

QUANTUM MECHANICS AND ATOMIC STRUCTURE

- **5.1** The Hydrogen Atom
- **5.2** Shell Model for Many-Electron Atoms
- **5.3** Aufbau Principle and Electron Configurations
- **5.4** Shells and the Periodic Table: Photoelectron Spectroscopy
- **5.5** Periodic Properties and Electronic Structure

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Colors of Fireworks

from atomic emission

red from Sr orange from Ca yellow from Na green from Ba blue from Cu

Key question 1: what is the quantum mechanical atomic model?

Key question 2: what is the origin of the periodicity of atomic properties?

5.1 THE HYDROGEN ATOM

- The hydrogen atom is the simplest example of a one-electron atom or ion. (i.e. He⁺, Li²⁺, ...)
- Schrödinger equation

$$
\left[\frac{-h^2}{8\pi^2 m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) + V(x, y, z)\right] \psi(x, y, z) = E\psi(x, y, z)
$$

- For a hydrogen atom, $V(r) =$ Coulomb potential energy

Energy Levels

Solutions of the Schrödinger equation

$$
E = E_n = -\frac{Z^2 e^4 m_e}{8 \epsilon_0^2 n^2 h^2} \qquad n = 1, 2, 3, ...
$$

$$
E_n = -\frac{Z^2}{n^2}
$$
 (rydberg) $n = 1, 2, 3, ...$

1 rydberg = 2.18×10^{-18} J

No angular dependence due to the isotropic symmetry

Principal quantum number n: indexing the individual energy levels.

Wave Functions

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However, the wave function has angular dependence

Quantization of the Angular Momentums

Angular momentum quantum number ℓ : from the quantization of L²

any integral value from 0 to n-1

$$
L2 = \ell(\ell + 1)\frac{h2}{4\pi2} \qquad \ell = 0, 1, ..., n-1
$$

value of ℓ 0 1 2 3
orbital type s 0 d f orbitals

i.e.) $n = 1, \ell = 0$: 1s state; $n = 4, \ell = 3$: 4f state;

 \triangleright **Magnetic quantum number m** from the quantization of L_z

$$
L_z = m \frac{h}{2\pi}
$$
 $m = -\ell, -\ell + 1, ..., \ell - 1, \ell$

$$
\qquad \qquad + of m = 2\ell + 1
$$

For n = 1 (the ground state), $(\ell = 0, m = 0)$

For $n = 2$, $n^2 = 4$ allowed sets $(\ell = 0, m = 0), (\ell = 1, m = 1), (\ell = 1, m = 0), (\ell = 1, m = -1)$

For every value of n , n^2 sets of quantum numbers

TABLE 5.1

Allowed Values of Quantum Numbers for One-Electron Atoms

- \triangleright Each set (n, ℓ, m) identifies a specific **quantum state** of the atom.
- \triangleright A total of n^2 specific quantum states correspond to the single E_{n} .

This set of state is to be **degenerate**.

Orbital

 \triangleright A wave function $\psi_{n\ell m}(r, \theta, \phi)$ for a one-electron atom in the state (n, ℓ, m) is called an **orbital**.

EXAMPLE 5.1

Give the names of all the orbitals with $n = 4$, and state how many m values correspond to each type of orbital.

$$
\ell = 3 \quad 4f \qquad \qquad # \text{ of } m = 7
$$

total n^2 = 16 orbitals

TABLE 5.2

Angular and Radial Parts of Wave Functions for One-Electron Atoms

 $\sqrt{5}$

Sizes and Shapes of Orbitals

$$
\psi_{n\ell m}(r,\theta,\phi)=R_{n\ell}(r)Y_{\ell m}(\theta,\phi)
$$

- Three spatial dimensions (r, θ, ϕ) with the value of wave function.
- **Graphical representation of the orbitals**
- 1) Slicing up 3D space into various 2D and 1D regions and examining the value of wave function at each point.

2) Looking only at the radial behavior. "vertical slice"

 $\psi_{100}(r, \theta, \phi)$

- $(\psi_{n\ell m})^2$, probability density of finding the electron in a small volume element dV

s orbitals

- 1s (n = 1, ℓ **= 0, m = 0)** \rightarrow R₁₀(r) and Y₀₀(θ , Φ)

a function of r only

- * spherically symmetric
- * exponential decaying
- * no nodes

$$
-2s
$$
 (n = 2, ℓ = 0, m = 0)

$$
\psi_{200}(r,\theta,\!\phi)=\!\frac{1}{4\sqrt{2\pi}}\!a_0^{-3/2}\!\!\left(2-\frac{r}{a_0}\right)\mathrm{e}^{-r/2a_0}
$$

zero at $r = 2a_0 = 1.06$ Å nodal sphere or radial node [r<2a_o: Ψ>0, positive] [r>2a_o: Ψ<0, negative]

p orbitals

 ρ **p orbitals**: Y_{em}(θ , ϕ) has separate lobes with positive and negative phase, with a node between them.

$$
- n = 2
$$
, $\ell = 1$, $m = 0 \rightarrow 2p_0$ orbital : $R_{21}Y_{10}$

$$
\psi_{210}(r,\theta,\!\phi)=\frac{1}{4\sqrt{2\pi}}\quad a_0^{-3/2}\frac{r}{a_0}\,\mathrm{e}^{-r/2a_0}\cos\theta
$$

- $\cdot \Phi = 0 \rightarrow$ cylindrical symmetry about the z-axis
- \cdot R₂₁(r) \rightarrow r/a₀ no radial nodes except at the origin
- \cdot cos θ \rightarrow angular node at θ = 90°, x-y nodal plane
- \cdot r cos $\theta \rightarrow$ z-axis $2p_0 \rightarrow$ labeled as $2p_z$

 $-n = 2$, $\ell = 1$, $m = \pm 1 \rightarrow 2p_{+1}$ and $2p_{-1}$

- \cdot Y₁₁(θ , Φ) \rightarrow e^{$\pm i\Phi$} = cos Φ \pm i sin Φ \leftarrow Euler's formula was used
- · taking linear combinations two real orbitals \rightarrow constructed 2p_x and 2p_y

(real functions using Euler's formula)

$$
2p_x = \frac{1}{\sqrt{2}} (2p_{+1} + 2p_{-1}) = \frac{1}{4\sqrt{2\pi}} a_0^{-3/2} \frac{r}{a_0} e^{-r/2a_0} \sin\theta \cos\phi
$$

$$
2p_y = \frac{1}{i\sqrt{2}} (2p_{+1} - 2p_{-1}) = \frac{1}{4\sqrt{2\pi}} a_0^{-3/2} \frac{r}{a_0} e^{-r/2a_0} \sin\theta \sin\phi
$$

- p_x and p_y differ from p_z only in the angular factors (orientations).

General Chemistry I

 \mathcal{X}

d orbitals

d-orbitals

 $l = 2$ & $m = 0$: d_{z^2} ℓ = 2 & linear combination of m = +2, +1, -1, -2 $\bm{d_{xy}}$, $\bm{d_{yz}}$, $\bm{d_{xz}}$, $\bm{d_{x^2-y^2}}$

- Each d orbital has **two angular nodes**. i.e.) d_{xy} orbital: nodal surfaces of the xz and yz planes
- The radial functions, $R_{n2}(r)$, have $n 3$ **radial nodes**, giving **n - 1 total nodes**.

Orbital Shapes and Sizes

- 1. For a given value of *ℓ*, and **increase in** *n* leads to an **increase in the average distance** of the electron from the nucleus, and therefore in the size of the orbital.
- 2. An orbital with quantum numbers n and ℓ has ℓ **angular nodes and** − ℓ − **radial nodes, giving a total of – 1 nodes**. An angular node is defined by a plane. A radial node is defined by a spherical surface. For a one-electron atom or ion, **the energy depends only on the number of nodes** - that is, on n but not ℓ or m . The energy increases as the number of nodes increases.
- 3. As *r* approaches 0, $\psi_{n\ell m}(r, \theta, \phi)$ vanishes for all orbitals except **s orbitals**; thus, only an electron in an s orbital can "penetrate to the nucleus," that is, have a finite probability of being found right at the nucleus.

 The **average value of the distance** of the electron from the nucleus in that orbital, $\bar{r}_{n\ell}$

Electron Spin

m^s , spin magnetic quantum number

- An electron has two spin states, as \uparrow (up) and \downarrow (down), or a and b.
- the values of m_s , only +1/2 and -1/2

$$
\bigotimes m_s = +1/2
$$

- discovered by Stern and Gerlach experiment (1926)

*Also called the azimuthal quantum number.

5.2 SHELL MODEL FOR MANY-ELECTRON ATOMS

- In many-electron atoms, Coulomb potential energy equals the sum of *nucleus-electron* attractions and *electron-electron* repulsions.
- No exact solutions of Schrödinger equation
- In a helium atom,

- r_1 = the distance of electron 1 from the nucleus
- $r₂$ = the distance of electron 2 from the nucleus
- r_{12} = the distance between the two electrons

The Shell Model of the Atom

For Li with $Z = 3$,

$$
V = \frac{Ze^2}{4\pi\varepsilon_0} \left(-\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} \right) + \frac{e^2}{4\pi\varepsilon_0} \left(\frac{1}{r_{12}} + \frac{1}{r_{13}} + \frac{1}{r_{23}} \right)
$$

Effective potential energy

Taking into account both the attractive electron-nuclear forces and the average of the repulsive force among the electrons.

$$
V_{\text{eff}}(r) = -\frac{Z_{\text{eff}}e^2}{4\pi\varepsilon_0 r}
$$

Shielding Effects

- Energy-level diagrams for many-electron atoms
	- 1) **The degeneracy of the p, d, and f orbitals is removed**, due to the non-isotropic potential \rightarrow angular dependency

The energy levels depend on both n and ℓ

- 2) Energy values are distinctly shifted from the values of corresponding H orbitals due to the strong attraction by nuclei with $Z > 1$.
- Each electron attracted by the nucleus, and repelled by the other electrons.
- \rightarrow **shielded** from the full nuclear attraction by the other electrons.

5.3 AUFBAU PRINCIPLE AND ELECTRON CONFIGURATIONS

Electronic configuration of the atom

a list of all its occupied orbitals, with the numbers of electrons that each one contains.

i.e. Li: 1s² 2s¹

Pauli exclusion principle

No two electrons in an atom can have the same set of four quantum numbers, $(n, \, \ell, \, m, \, m_s)$.

or

Each atomic orbital with (n, ℓ, m) holds at most two electrons, one with spin up and the other with spin down.

Hund's rule

When electrons are added to orbitals of equal energy, a single electron enters each orbital before a second one enters any orbital.

In addition, the lowest energy configuration is the one with parallel spins.

Be, $1s^22s^2$ or [He] $2s^2$

 \blacksquare B, 1s²2s²2p¹ or [He]2s²2p¹

• C, $1s^22s^22p^2$ or [He] $2s^22p^2$

- **Magnetic properties** for a test of electronic configurations
	- **paramagnetic**: a substance attracted into a magnetic field with one or more unpaired electrons
	- **diamagnetic**: a substance pushed out of a magnetic field with all electrons paired
	- i.e.) paramagnetic: H, Li, B, C diamagnetic: He, Be

- Elements in **Period 2** (from Li to Ne), the valence shell with $n = 2$.
- **p-block elements**: B to Ne, filling of p orbitals **s-block elements**: H to Be, filling of s orbitals

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- **n = 3**: Na, [He]2s²2p⁶3s¹ or [Ne]3s¹ to Ar, [Ne]3s²3p⁶
- **n** $=$ **4**: from Sc (scandium, $Z = 21$) to Zn (zinc, $Z = 30$) the next 10 electrons enter the 3d-orbitals. **d-block elements**

The (n+ℓ**) rule**

Order of filling subshells in neutral atoms is determined by filling those with the lowest values of $(n+\ell)$ first. Subshells in a group with the same value of $(n+\ell)$ are filled in the order of increasing n, due to the orbital screening.

order: 1s < 2s < 2p < 3s < 3p < 4s < 3d $< 4p < 5s < 4d < \dots$

 n = 5: 5s-electrons followed by the 4delectrons **n = 6**: Ce (cerium, [Xe]4f¹5d¹6s²)

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Anomalous Configurations

Exceptions to the Aufbau principle Cr, predicted: $4s^23d^4$ \uparrow \uparrow \uparrow \uparrow Cr, observed: $4s^13d^5$ Copyright © 2006 Pearson Education, Inc., Publishing as Benjamin Cummings

Anomalous electron configurations

General Chemistry I

CHEM

General Chemistry I

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All atoms in a given **period** have the same type of core with the same n.

All atoms in a given **group** have analogous valance electron configurations that differ only in the value of n.

General Chemistry I

CHEMISTRY

5.4 SHELLS AND THE PERIODIC TABLE: PHOTOELECTRON SPECTROSCOPY

 A **shell** is defined precisely as a set of orbitals that have the same principal quantum number.

Photoelectron spectroscopy (PES)

determining the energy level of each orbital by measuring the ionization energy required to remove each electron from the atom

$$
IE = h\nu - \frac{1}{2} m_e v_{electron}^2
$$

- For Ne excited by X-rays with $v = 9.890 \times 10^{-10}$ m,

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General Chemistry I

- Koopmans's approximation,

 $I E_{\alpha} = -\epsilon_{\alpha}$

- with the frozen orbital approximation

The orbital energies are the same in the ion, despite the loss of an e.

i.e.) for Ne with $1s² 2s² 2p⁶$

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5.5 PERIODIC PROPERTIES AND ELECTRONIC STRUCTURE

- **Atomic radius**: defined as half the distance between the centers of neighboring atoms
- **Ionic radius**: its share of the distance between neighboring ions in an ionic solid

General trends

- **r decreases from left to right across a period** (effective nuclear charge increases)
- **r increases from top to bottom down a group** (change in n and size of valence shell)

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- The rate of increase changes considerably.

i.e.) Li⁺, Na⁺, to K⁺ substantial change to Rb⁺ , Cs⁺ small change due to filling d-orbitals

 \triangleright Lanthanide contraction filling of the 4f orbitals

 \triangleright **Molar volumes** (cm³ mol⁻¹) of atoms in the solid phase

= **size of the atoms** + geometry of the bonding

Periodic Trends in Ionization Energies

- From left to right, generally increase in IE_1 due to the increase of Z_{eff}
- From top to bottom, generally decrease in E_1 due to the increase of n
- **From He to Li, a large reduction in IE**₁ 2s e⁻ father than 1s e⁻, and 2s e⁻ sees a net +1 charge
- **From Be to B, slight reduction in IE** fifth e⁻ in a higher energy 2p orbital
- **From N to O, slight reduction in IE**₁ 2 e in the same 2p orbital leading to greater repulsion

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Electron Affinity

- The periodic trends in EA parallel those in IE_1 with one unit lower shift i.e.) F to F⁻, large EA because of its closed-shell configuration

⁵² **Key question 1: what is the quantum mechanical atomic model?**

Key question 2: what is the origin of the periodicity of atomic properties?

Comparison of atomic models

(A) The Classical Model (B) Bohr's Model (C) The Quantum Mechanics Model

For Chapter 5,

- Problem Sets
	- : 22, 28, 32, 48, 56
- Chapter Summary (Choose one)
	- : Valence electrons, Solution for multielectron system

