

QUANTUM MECHANICS AND ATOMIC STRUCTURE

- 5.1 The Hydrogen Atom
- 5.2 Shell Model for Many-Electron Atoms
- **5.3** Aufbau Principle and Electron Configurations
- 5.4 Shells and the Periodic Table: Photoelectron Spectroscopy
- **5.5** Periodic Properties and Electronic Structure



Colors of Fireworks

from atomic emission

red from Sr orange from Ca yellow from Na green from Ba

blue from Cu



Key question 1: what is the quantum mechanical atomic model?



Key question 2: what is the origin of the periodicity of atomic properties?





5.1 THE HYDROGEN ATOM

- The hydrogen atom is the simplest example of a one-electron atom or ion. (i.e. He⁺, Li²⁺, ...)
- Schrödinger equation

$$\left[\frac{-h^2}{8\pi^2 m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) + V(x, y, z)\right]\psi(x, y, z) = E\psi(x, y, z)$$

- For a hydrogen atom, V(r) = Coulomb potential energy







Energy Levels

> Solutions of the Schrödinger equation

$$E = E_n = -\frac{Z^2 e^4 m_e}{8\epsilon_0^2 n^2 h^2}$$
 $n = 1, 2, 3, ...$

$$E_n = -\frac{Z^2}{n^2}$$
 (rydberg) $n = 1, 2, 3, ...$

1 rydberg = 2.18×10^{-18} J

No angular dependence due to the isotropic symmetry

Principal quantum number n: indexing the individual energy levels.





Wave Functions

However, the wave function has angular dependence



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Quantization of the Angular Momentums

> Angular momentum quantum number ℓ : from the quantization of L²

any integral value from 0 to n-1

$$L^{2} = \ell(\ell + 1) \frac{h^{2}}{4\pi^{2}} \qquad \ell = 0, 1, ..., n-1$$
value of ℓ 0 1 2 3
orbital type s p d f orbitals

i.e.) n = 1, $\ell = 0$: 1s state; n = 4, $\ell = 3$: 4f state;

> Magnetic quantum number m : from the quantization of L_z

$$L_{z} = m \frac{h}{2\pi} \qquad m = -\ell, -\ell + 1, ..., \ell - 1, \ell$$
$$\# of m = 2\ell + 1$$



For n = 1 (the ground state), $(\ell = 0, m = 0)$

For n = 2, $n^2 = 4$ allowed sets ($\ell = 0, m = 0$), ($\ell = 1, m = 1$), ($\ell = 1, m = 0$), ($\ell = 1, m = -1$)

For every value of n, n² sets of quantum numbers

TABLE 5.1

Allowed Values of Quantum Numbers for One-Electron Atoms

n	1		2	3					
ℓ	0	0	1	0	1	2			
m	0	0	-1, 0, +1	0	-1, 0, +1	-2, -1, 0, +1, +2			
Number of degenerate states for each ℓ	1	1	3	1	3	5			
Number of degenerate states for each <i>n</i>	1		4			9			



- Each set (n, l, m) identifies a specific quantum state of the atom.
- A total of n² specific quantum states correspond to the single E_n.

This set of state is to be **degenerate**.

ŀ	E				9
п					
	$\ell = 0$	$\ell = 1$	$\ell = 2$	$\ell = 3$	E = 0
3	3s [1]	3p [3]	3 <i>d</i> [5]		
2	2s [1]	<u>2p</u> [3]			
1	1s [1]				

Orbital

A wave function $\psi_{n\ell m}(r,\theta,\phi)$ for a one-electron atom in the state (n, ℓ, m) is called an **orbital**.

EXAMPLE 5.1

Give the names of all the orbitals with n = 4, and state how many m values correspond to each type of orbital.

n = 4	ℓ=0 4s	# of <i>m</i> = 1

$$\ell = 1$$
 4p # of $m = 3$

 $\ell = 2$ 4d # of m = 5

$$\ell = 3$$
 4f $\#$ of $m = 7$

total $n^2 = 16$ orbitals



TABLE 5.2

Angular and Radial Parts of Wave Functions for One-Electron Atoms



Sizes and Shapes of Orbitals

$$\psi_{n\ell m}(r,\theta,\phi) = R_{n\ell}(r)Y_{\ell m}(\theta,\phi)$$

- Three spatial dimensions (r, θ, ϕ) with the value of wave function.

Graphical representation of the orbitals

1) Slicing up 3D space into various 2D and 1D regions and examining the value of wave function at each point.



2) Looking only at the radial behavior. "vertical slice"



 $\psi_{100}(r, \theta, \phi)$



- $(\psi_{n\ell m})^2$, probability density of finding the electron in a small volume element dV



s orbitals



- 1s (n = 1, ℓ = 0, m = 0) $\rightarrow R_{10}(r)$ and $Y_{00}(\theta, \Phi)$



a function of r only

- * spherically symmetric
- * exponential decaying
- * no nodes

$$\psi_{200}(r,\theta,\phi) = \frac{1}{4\sqrt{2\pi}} a_0^{-3/2} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$$

zero at $r = 2a_0 = 1.06$ Å nodal sphere or radial node [r<2a_o: Ψ >0, positive] [r>2a_o: Ψ <0, negative]







p orbitals

> **p orbitals**: $Y_{\ell m}(\theta, \phi)$ has separate lobes with positive and negative phase, with a node between them.

- n = 2,
$$\ell$$
 =1, m = 0 \rightarrow 2p₀ orbital : R₂₁ Y₁₀

$$\psi_{210}(r,\theta,\phi) = \frac{1}{4\sqrt{2\pi}} \quad a_0^{-3/2} \frac{r}{a_0} e^{-r/2a_0} \cos\theta$$

- $\cdot \Phi = 0 \rightarrow$ cylindrical symmetry about the z-axis
- $\cdot R_{21}(r) \rightarrow r/a_0$ no radial nodes except at the origin
- $\cos \theta \rightarrow$ angular node at $\theta = 90^{\circ}$, x-y nodal plane
- · r cos $\theta \rightarrow z$ -axis 2p₀ \rightarrow labeled as 2p_z



- n = 2, $\ell = 1$, m = $\pm 1 \rightarrow 2p_{\pm 1}$ and $2p_{\pm 1}$

- $\cdot Y_{11}(\theta, \Phi) \rightarrow e^{\pm i\Phi} = \cos \Phi \pm i \sin \Phi \leftarrow$ Euler's formula was used
- taking linear combinations two real orbitals \rightarrow constructed $2p_x$ and $2p_y$

(real functions using Euler's formula)

$$2p_x = \frac{1}{\sqrt{2}} \left(2p_{+1} + 2p_{-1}\right) = \frac{1}{4\sqrt{2\pi}} a_0^{-3/2} \frac{r}{a_0} e^{-r/2a_0} \sin\theta \cos\phi$$

$$2p_y = \frac{1}{i\sqrt{2}} \left(2p_{+1} - 2p_{-1}\right)_{,} = \frac{1}{4\sqrt{2\pi}} a_0^{-3/2} \frac{r}{a_0} e^{-r/2a_0} \sin\theta \sin\phi$$

- p_x and p_y differ from p_z only in the angular factors (orientations).







General Chemistry I

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d orbitals

d-orbitals

 $\ell = 2 \& m = 0: \ \mathbf{d}_{z^2}$ $\ell = 2 \&$ linear combination of $m = +2, +1, -1, -2 \ \mathbf{d}_{xy}, \mathbf{d}_{yz}, \mathbf{d}_{xz}, \mathbf{d}_{x^2-y^2}$

- Each d orbital has two angular nodes.
 i.e.) d_{xy} orbital: nodal surfaces of the xz and yz planes
- The radial functions, $R_{n2}(r)$, have n 3 radial nodes, giving n - 1 total nodes.









Orbital Shapes and Sizes

- For a given value of ℓ, and increase in n leads to an increase in the average distance of the electron from the nucleus, and therefore in the size of the orbital.
- 2. An orbital with quantum numbers n and l has l angular nodes and n l 1 radial nodes, giving a total of n 1 nodes. An angular node is defined by a plane. A radial node is defined by a spherical surface. For a one-electron atom or ion, the energy depends only on the number of nodes that is, on n but not l or m. The energy increases as the number of nodes increases.
- 3. As *r* approaches 0, $\psi_{n\ell m}(r, \theta, \phi)$ vanishes for all orbitals except **s** orbitals; thus, only an electron in an s orbital can "penetrate to the nucleus," that is, have a finite probability of being found right at the nucleus.



> The average value of the distance of the electron from the nucleus in that orbital, $\bar{r}_{n\ell}$



Electron Spin

➢ m_s, spin magnetic quantum number

- An electron has two spin states, as ↑(up) and ↓(down), or a and b.
- the values of m_s , only +1/2 and -1/2



- discovered by Stern and Gerlach experiment (1926)

Name	Symbol	Values	Specifies	Indicates
principal	п	1, 2,	shell	size
orbital angular	l	$0, 1, \ldots, n-1$	subshell:	shape
momentum*			$l = 0, 1, 2, 3, 4, \ldots$	
			s, p, d, f, g,	
magnetic	m_l	$l, l-1, \ldots, -l$	orbitals of subshell	orientation
spin magnetic	m_s	$+\frac{1}{2}, -\frac{1}{2}$	spin state	spin direction

*Also called the azimuthal quantum number.



5.2 SHELL MODEL FOR MANY-ELECTRON ATOMS

- In many-electron atoms, Coulomb potential energy equals the sum of *nucleus-electron* attractions and *electron-electron* repulsions.
- No exact solutions of Schrödinger equation
- In a helium atom,

Attraction of Attraction
electron 1 to electron 2 to the nucleus the nucleus
$$V = -\frac{2e^2}{4\pi\epsilon_0 r_1} - \frac{2e^2}{4\pi\epsilon_0 r_2} + \frac{e^2}{4\pi\epsilon_0 r_{12}}$$

- r₁ = the distance of electron 1 from the nucleus
- r₂ = the distance of electron 2 from the nucleus
- r₁₂ = the distance between the two electrons



The Shell Model of the Atom

For Li with
$$Z = 3$$
,

$$\mathsf{V} = \frac{Ze^2}{4\pi\varepsilon_0} \left(-\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} \right) + \frac{e^2}{4\pi\varepsilon_0} \left(\frac{1}{r_{12}} + \frac{1}{r_{13}} + \frac{1}{r_{23}} \right)$$



Effective potential energy

Taking into account both the attractive electron-nuclear forces and the average of the repulsive force among the electrons.

$$V_{\rm eff}(r) = -\frac{Z_{eff}e^2}{4\pi\varepsilon_0 r}$$



Shielding Effects

- Energy-level diagrams for many-electron atoms
 - The degeneracy of the p, d, and f orbitals is removed, due to the non-isotropic potential → angular dependency

The energy levels depend on both n and ℓ

- 2) Energy values are distinctly shifted from the values of corresponding H orbitals due to the strong attraction by nuclei with Z > 1.
- Each electron attracted by the nucleus, and repelled by the other electrons.
- \rightarrow shielded from the full nuclear attraction by the other electrons.





5.3 AUFBAU PRINCIPLE AND ELECTRON CONFIGURATIONS

Electronic configuration of the atom

a list of all its occupied orbitals, with the numbers of electrons that each one contains.

i.e. Li: 1s² 2s¹

Pauli exclusion principle

No two electrons in an atom can have the same set of four quantum numbers, (n, ℓ, m, m_s) .

or

Each atomic orbital with (n, ℓ, m) holds at most two electrons, one with spin up and the other with spin down.



Hund's rule

When electrons are added to orbitals of equal energy, a single electron enters each orbital before a second one enters any orbital.

In addition, the lowest energy configuration is the one with parallel spins.





Be, 1s²2s² or [He]2s²

2s	$\uparrow\downarrow$	
1s	$\uparrow\downarrow$	

B, 1s²2s²2p¹ or [He]2s²2p¹



• C, $1s^22s^22p^2$ or [He] $2s^22p^2$ $\rightarrow 1s^22s^22p_x^{-1}2p_y^{-1}$





- > Magnetic properties for a test of electronic configurations
 - paramagnetic: a substance attracted into a magnetic field with one or more unpaired electrons
 - diamagnetic: a substance pushed out of a magnetic field with all electrons paired
 - i.e.) paramagnetic: H, Li, B, C diamagnetic: He, Be





- Elements in **Period 2** (from Li to Ne), the valence shell with n = 2.
- p-block elements: B to Ne, filling of p orbitals
 s-block elements: H to Be, filling of s orbitals



		1 <i>s</i>	2 <i>s</i>	$2p_x$	$2p_y$	$2p_z$	
	H: 1 <i>s</i> ¹	_1					
	He: 1 <i>s</i> ²	1					
	Li: 1 <i>s</i> ² 2 <i>s</i> ¹	11	1				
	Be: $1s^2 2s^2$	11	_1/				
	B: $1s^2 2s^2 2p_x^1$	11	1,	1			
	C: $1s^2 2s^2 2p_x^1 2p_y^1$	_1レ_	_1/	_1	_1		
	N: $1s^2 2s^2 2p_x^1 2p_y^1 2p_z^1$	_1レ_	11/	_1	_1	_1	
	O: $1s^2 2s^2 2p_x^2 2p_y^1 2p_z^1$	_1L	<u> 1 </u>	_11	_1	_1	
	F: $1s^2 2s^2 2p_x^2 2p_y^2 2p_z^1$	_1L	<u> 1l </u>	_1L	11	_1	
General Ch	Ne: $1s^2 2s^2 2p_x^2 2p_y^2 2p_z^2$	11	_1/_	_11	_1/	_11	HEMISTRY

- **n = 3**: Na, [He]2s²2p⁶3s¹ or [Ne]3s¹ to Ar, [Ne]3s²3p⁶
- n = 4: from Sc (scandium, Z = 21) to Zn (zinc, Z = 30) the next 10 electrons enter the 3d-orbitals. d-block elements

> The ($n+\ell$) rule

Order of filling subshells in neutral atoms is determined by filling those with the lowest values of $(n+\ell)$ first. Subshells in a group with the same value of $(n+\ell)$ are filled in the order of increasing n, due to the orbital screening. ^{*E*}

order: 1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d < ...

 n = 5: 5s-electrons followed by the 4delectrons
 n = 6: Ce (cerium, [Xe]4f¹5d¹6s²)



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Anomalous Configurations

Anomalous electron configurations^{*}

Period	Ζ	Element	Configuration	Period	Ζ	Element	Configuration
4	24	Cr	$[Ar]4s^13d^5$	6	57	La	$[Xe]6s^25d^1$
4	29	Cu	$[Ar]4s^{1}3d^{10}$	6	58	Ce	$[Xe]6s^24f^15d^1$
5	41	Nb	$[Kr]5s^{1}4d^{4}$	6	64	Gd	$[Xe]6s^24f^75d^1$
5	42	Mo	$[Kr]5s^{1}4d^{5}$	6	78	Pt	$[Xe]6s^{1}4f^{14}5d^{9}$
5	44	Ru	$[Kr]5s^{1}4d^{7}$	6	79	Au	$[Xe]6s^{1}4f^{14}5d^{10}$
5	45	Rh	$[Kr]5s^{1}4d^{8}$	7	89	Ac	$[\mathrm{Rn}]7s^26d^1$
5	46	Pd	$[Kr]4d^{10}$	7	90	Th	$[Rn]7s^26d^2$
5	47	Ag	$[Kr]5s^{1}4d^{10}$	7	91	Pa	$[\mathrm{Rn}]7s^25f^26d^1$
				7	92	U	$[\mathrm{Rn}]7s^25f^36d^1$
							KAISI

General Chemistry I

CHFM

1 <i>s</i>																	1 <i>s</i>	
Н																	He	
2 <i>s</i> –fi	illing													2 <i>p</i> -f	illing			
Li	Be											В	С	N	0	F	Ne	
													1020					
3s-fi	illing													3 <i>p</i> -f	illing			
Na	Mg											Al	Si	Р	S	Cl	Ar	
	Ū																Contractor -	
4 <i>s</i> –fi	illing					3 <i>d</i> -f	illing					4 <i>p</i> -filling						
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Со	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr	
					$3d^{5}4s^{1}$					$3d^{10}4s^1$								
5 <i>s</i> –fi	illing		10 - 11 - 11 - 11 - 11 - 11 - 11 - 11 -	P		4 <i>d</i> -f	illing			AI		5 <i>p</i> -filling						
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	Ι	Xe	
				$4d^{4}5s^{1}$	$4d^{5}5s^{1}$		$4d^{7}5s^{1}$	$4d^{8}5s^{1}$	$4d^{10}$	$4d^{10}5s^{1}$		_						
6s–fi	illing		an a			5 <i>d</i> –f	illing							6 <i>p</i> –f	illing			
Cs	Ba	Lu	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	T1	Pb	Bi	Po	At	Rn	
									$5d^96s^1$	$5d^{10}6s^{1}$	U							
7 <i>s</i> –fi	illing					6 <i>d</i> –f	illing								-			
Fr	Ra	Lr	Rf	Ha	Sg	Ns	Hs	Mt	Uun	Uuu								
	3000000	and a second			0	Cretorinizi												

	4 <i>f</i> –filling													
La 5d ¹ 6	a	$\frac{\text{Ce}}{4f^45d^46s^2}$	Pr	Nd	Pm	Sm	Eu	$Gd_{4\vec{f}5d^16s^2}$	Tb	Dy	Ho	Er	Tm	Yb

	5 <i>f</i> –filling													
Ac 6d ¹ 7s ²	$_{6d^27s^2}$	$\frac{\text{Pa}}{5f^26d^17s^2}$	U 5f ³ 6d ¹ 7s ²	$\frac{Np}{5f^{4}6d^{1}7s^{2}}$	Pu	Am	$Cm_{5\vec{f}6d^17s^2}$	Bk	Cf	Es	Fm	Md	No	
strv	1									CH	EMIS	TRY	>	

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All atoms in a given **period** have the same type of core with the same n.

All atoms in a given group have analogous valance electron configurations that differ only in the value of n.



5.4 SHELLS AND THE PERIODIC TABLE: PHOTOELECTRON SPECTROSCOPY

A shell is defined precisely as a set of orbitals that have the same principal quantum number.

Photoelectron spectroscopy (PES)

determining the energy level of each orbital by measuring the ionization energy required to remove each electron from the atom

$$\mathbf{IE} = \mathbf{h}\boldsymbol{\nu} \ -\frac{1}{2} \ \boldsymbol{m}_e \boldsymbol{v}_{electron}^2$$





- For Ne excited by X-rays with $\nu = 9.890 \times 10^{-10}$ m,





- Koopmans's approximation,

 $IE_{\alpha} = -\epsilon_{\alpha}$

- with the frozen orbital approximation

The orbital energies are the same in the ion, despite the loss of an e⁻.

i.e.) for Ne with $1s^2 2s^2 2p^6$





5.5 PERIODIC PROPERTIES AND ELECTRONIC STRUCTURE

- Atomic radius: defined as half the distance between the centers of neighboring atoms
- Ionic radius: its share of the distance between neighboring ions in an ionic solid

General trends

- r decreases from left to right across a period (effective nuclear charge increases)
- r increases from top to bottom down a group

(change in n and size of valence shell)







- The rate of increase changes considerably.
 - i.e.) Li⁺ ,Na⁺, to K⁺ substantial change to Rb⁺, Cs⁺ small change due to filling d-orbitals
- Lanthanide contraction filling of the 4f orbitals



> Molar volumes (cm³ mol⁻¹) of atoms in the solid phase

= size of the atoms + geometry of the bonding

H 11.4																	He 21.0
Li 13.0	Be 4.85											B 4.39	C 3.42	N 13.5	0 17.4	F 11.2	Ne 13.2
Na 23.8	Mg 14.0											Al 10.0	Si 12.1	Р 17.0	S 15.5	Cl 17.4	Ar 22.6
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
45.9	26.2	15.0	10.6	8.32	7.23	7.35	7.09	6.67	6.59	7.11	9.16	11.8	13.6	13.0	16.4	19.8	28.0
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Те	I	Xe
55.8	33.9	19.9	14.0	10.8	9.38	8.63	8.17	8.28	8.56	10.3	13.0	15.8	16.3	18.2	20.5	25.7	35.9
Cs	Ba	Lu	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po		Rn
70.9	38.2	17.8	13.4	10.9	9.47	8.86	8.42	8.52	9.09	10.2	14.1	17.2	18.3	21.3	23.0		50.5



Periodic Trends in Ionization Energies

- From left to right, generally increase in IE₁ due to the increase of Z_{eff}
- From top to bottom, generally decrease in IE₁ due to the increase of n
- From He to Li, a large reduction in IE₁
 2s e⁻ father than 1s e⁻, and 2s e⁻ sees a net +1 charge
- From Be to B, slight reduction in IE₁
 fifth e⁻ in a higher energy 2p orbital
- From N to O, slight reduction in IE₁
 2 e⁻ in the same 2p orbital leading to greater repulsion





Ge

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Electron Affinity

- The periodic trends in EA parallel those in IE₁ with one unit lower shift i.e.) F to F⁻, large EA because of its closed-shell configuration

H 73																	He *
Li 60	Be *											В 27	C 122	N *	0 141	F 328	Ne *
Na 53	Mg ⊁								(kJ ma	ol⁻¹)	Al 43	Si 134	Р 72	S 200	Cl 349	Ar *
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
48	2	18	8	51	64	⊁	16	64	111	118	*	29	116	78	195	325	*
Rb	Sr	Y	Zr	Nb	Mo	Тс	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
47	5	30	41	86	72	53	99	110	52	126	*	29	116	103	190	295	*
Cs	Ba	Lu	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Ро	At	Rn
46	14	50	*	31	79	14	106	151	214	223	*	19	35	91	183	270	*



Key question 1: what is the quantum mechanical atomic model?



Key question 2: what is the origin of the periodicity of atomic properties?





Comparison of atomic models

(A) The Classical Model



(B) Bohr's Model



(C) The Quantum Mechanics Model





For Chapter 5,

- Problem Sets
 - : 22, 28, 32, 48, 56
- Chapter Summary (Choose one)
 - : Valence electrons, Solution for multielectron system

